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AIR SHOCK HURLING OF AN UNFASTENED SOLID NEAR A FLAT OBSTACLE

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A formulation is given of the problem of a shock hurling a body near a solid flat obstacle. It is considered that the condition of a long shock interacting with the body [1] is satisfied and the force pattern is representable by two phases, diffraction and quasistationary streamlining. Initial conditions for the origination of different modes of motion, as well as the transient conditions associated with a variable mode during shock interaction with the obstacle, are considered for two versions of the diffraction load representation. The solution is obtained by a numerical method.

1. A body is considered that has a plane Ω of material symmetry in which forces from a shock and the reactions of unilateral constraints act, which corresponds to the plane-parallel motion of the body with variable (from 1 to 3) degrees of freedom. Let the plane Ω coincide with the inertial XOY coordinate system with origin at the body center of mass, which is symmetric relative to Y and with two points of contact with the obstacle for $t \le 0$ (Fig. 1). The shock is propagated along the X axis and is continuous with the body at $t = 0$. The unperturbed wave parameters are associated with the point $X = 0$.

It is assumed that the system of forces in the diffraction phase is independent of the body displacements, which are not substantial, while it is determined in the streamlining phase by stationary aerodynamics relationships in which the time t is a parameter [1]. Collision of the body support with the obstacle is considered absolutely inelastic while the resistance to displacement is subject to Coulomb's law. Four modes of motion are possible: 1) ($E = 1$) rotation in combination with slip along the obstacle; 2) ($E = 2$) rotation around a fixed axis; 3) ($E = 3$) slip; 4) ($E = 4$) flight without contact with the obstacle. Mode alternation is allowable during the motion. The criterion $E = 0$ is introduced for the state of rest.

2. The load in the diffraction phase can be approximated by an instantaneous impulse or function of time, which is important to estimation of the body acceleration during its most intensive loading. It is considered that the impulses S_W, S_A, and the moment of the impulse M_S are known along the X, Y axes. We then write the approximate expressions for the frontal force W, the lift force A, and the moment M_0 for the second case.

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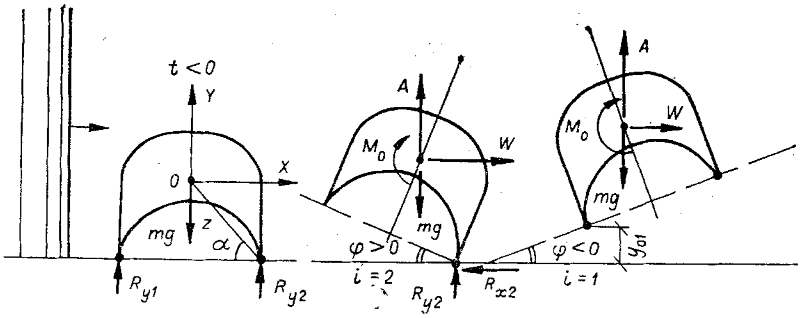

Fig. 1

$$W(t) = W_m(1 - t/\tau_0) \quad (0 < t \leqslant \tau < \tau_0),$$
$$A(t) = A_m t/\tau_1 \; (0 < t \leqslant \tau_1/2), \quad A(t) = A'_m(1 - t/\tau_A)\,(\tau_1 < t \leqslant \tau),$$
$$M_0 = M_m(1 - t/\tau_M) \quad (0 < t \leqslant \tau),$$

in which the constants are determined in terms of the impulse components taking into account the connection t = τ with the load functions in the quasistationary streamlining phase to the end of the diffraction phase

$$W_m = K_f F_\delta \Delta p_{\text{ref}} \quad A_m = 2S_A/\tau - (1/2)\bar{A}, \quad A'_m = \bar{A}\tau_A/(\tau_A - \tau),$$
$$M_m = \begin{cases} (2M_S - \tau\bar{M}_0)/\tau \quad (M_S\bar{M}_0 > 0), \\ \dfrac{M_S}{\tau} - \operatorname{sgn}\bar{M}_0 \sqrt{\left(\dfrac{M_S}{\tau}\right)^2 - \left(\dfrac{2M_S}{\tau} + \bar{M}_0\right)\bar{M}_0} \quad (M_S\bar{M}_0 < 0,\; M_S^* \equiv \\ \equiv |M_S/(\bar{M}_0\tau)| \geqslant \sqrt{2} - 1). \end{cases}$$

The condition $M^*_S \geq \sqrt{2} - 1$ denotes no substantial difference in the eccentricities of the aerodynamic (h°) and the impulse (h).

For the second phase, under the assumption that the Reynolds numbers Re are in the post-critical domain, i.e., self-similarity in Re holds, as is characteristic for poorly streamlined bodies, the loads are determined by the relationships

$$W = \Delta p_{\text{vel}} f(\Delta p_f, t) l^2 \hat{C}_W(\varphi, y)\alpha_W(\varphi, y, \mathrm{M}(t)),$$
$$A = \Delta p_{\text{vel}} f(\Delta p_f, t) l^2 \hat{C}_A(\varphi, y)\alpha_A(\varphi, y, \mathrm{M}(t)),$$
$$M_0 = \Delta p_{\text{vel}} f(\Delta p_f, t) l^3 \hat{C}_M(\varphi, y)\alpha_M(\varphi, y, \mathrm{M}(t)),$$

where Δp_f, Δp_{vel}, and Δp_{ref} are the pressure, the velocity head on the wave front, and the pressure of reflection from the solid wall, f is the function of velocity-head weakening $[f(\Delta p_f, 0) = 1]$; $\hat{C}_{W,A,M}$ are functions for the aerodynamic coefficients that depend on the generalized body coordinates (φ, y) for the Mach number M << 1; $\alpha_{W,A,M}$ are corrections for compressibility of air; F_δ is the middle; $K_f \leq 1$ is the mode factor; and l is the characteristic body dimension.

The time τ is determined from the condition of connection of expressions for the function W by the solution of the equation

$$W(\tau) \equiv W_m(1 - \tau/\tau_0) = Q(\tau),$$
$$Q(t) \equiv \Delta p_{\text{vel}} f(\Delta p_f, t) l^2 C_W(0, 0)\alpha_W(0, 0, \mathrm{M}(t)), \quad \tau_0 = (1/2)\tau(1 - S_W/(W_m\tau)).$$

From the condition of continuity for A at t = τ, t = τ_1, as well as for M_0 at t = τ, we have (M_f is the Mach number on the shock front)

$$\tau_A = (A_m - (1/2)\bar{A})/(A_m - \bar{A}), \quad \bar{A} = \Delta p_{\text{vel}} l^2 \hat{C}_A(0, 0)\alpha_A(0, 0, \mathrm{M}(0)),$$
$$\tau_M = M_m\tau/(M_m - \bar{M}_0), \quad \bar{M}_0 = \Delta p_{\text{vel}} l^3 \hat{C}_M(0, 0)\alpha_M(0, 0, \mathrm{M}(0)), \quad \mathrm{M}(0) = \mathrm{M}_f.$$

The functions $\hat{C}$ and α are obtained by the methods of experimental aerodynamics, and f and M from the solution of the problem of a point explosion [2]. The impulses S_W, S_A, and S_M are measured by using dynamic scales [3, 4].

3. The behavior of a body under a load is described by dynamic equilibrium equations and equations of constraints. The equations of plane motion of a body in projections on the X, Y axes have the form

$$mX^{\cdot\cdot} = \sum_j F_{xj},\ mY^{\cdot\cdot} = \sum_j F_{yj},\ I\varphi^{\cdot\cdot} = \sum_j \mathrm{mom}_z F_j, \tag{3.1}$$

where φ is the angle of body rotation around a center; m, body mass; I, its central moment of inertia relative to the z axis; $\sum_j F_{xj}, \sum_j F_{yj}$, projections of the principal active and reactive force vectors; $\sum_j \mathrm{mom}_z F_j$, moment of the principal vector relative to the z axis. The dot denotes the derivative with respect to t.

In the general case we represent the reactions of the constraints in the form of (nonnegative) vertical R_{yi} and horizontal R_{xi} components ($i = 1, 2$). For E = 1 a constraint maintaining one of the supports on the obstacle is imposed on the body. We determine the nonzero reactions R_{xi} and R_{yi} by using the subscript i:

$$i = 1\ (\mathrm{sgn}\,\varphi = -1),\ i = 2\ (\mathrm{sgn}\,\varphi = 1). \tag{3.2}$$

Denoting the possible displacements of the reference points by x_i and y_i, we have the equation of the constraint

$$y_i = 0 \tag{3.3}$$

that yields the relationship $Y(\varphi)$.

For a body with two degrees of freedom, we have the equations of motion and the relationship for the vertical reaction from (3.1)

$$I\varphi^{\cdot\cdot} = M_0 + z(R_{xi}\sin\gamma - R_{yi}\cos\gamma)\,\mathrm{sgn}\,\varphi,\ \gamma = \varphi + \alpha\,\mathrm{sgn}\,\varphi,\ \alpha < \pi/2; \tag{3.4}$$

$$mX^{\cdot\cdot} = W - R_{xi}; \tag{3.5}$$

$$R_{yi} = mY^{\cdot\cdot} - \hat{A},\ \hat{A} = A - mg. \tag{3.6}$$

We write for the horizontal reaction

$$|R_{xi}| = \mu_0 R_{yi}, \tag{3.7}$$

where z is the reversal radius; α, angular coordinate of the center of inertia; μ_0, friction coefficient; and g, free-fall acceleration.

Taking account of the sign of the reference point velocity $\dot{x}_i$, we have a system of equations for the case $E = 1\ (\varphi \neq 0,\ x_i^{\cdot} \neq 0,\ R_{yi} > 0),\ \beta = mz^2/I$ from (3.3)-(3.7)

$$\begin{gathered}\varphi^{\cdot\cdot} = f_1(M_0, \hat{A}, \varphi^{\cdot}, \gamma, \mu),\\ f_1 = [M_0 + (1/2)\beta I\varphi^{\cdot 2}(\sin 2\gamma - 2\mu\sin^2\gamma)\\ - \hat{A}z(\mu\sin\gamma - \cos\gamma)\,\mathrm{sgn}\,\varphi][1 - (1/2)\beta(\mu\sin 2\gamma - 2\cos^2\gamma)]^{-1}/I,\\ X^{\cdot\cdot} = W/m - \mu[-\hat{A}/m + z(f_1(M_0, \hat{A}, \varphi^{\cdot}, \gamma, \mu)\cos\gamma - \varphi^{\cdot 2}\sin\gamma)\,\mathrm{sgn}\,\varphi],\\ x_i^{\cdot} = X^{\cdot} z\varphi^{\cdot}\sin\gamma\,\mathrm{sgn}\,\varphi,\quad Y = z(\sin\gamma\,\mathrm{sgn}\,\varphi - \sin\alpha),\\ R_{yi} = -\hat{A} + mz[f_1(M_0, \hat{A}, \varphi^{\cdot}, \gamma, \mu)\cos\gamma - \varphi^{\cdot 2}\sin\gamma]\,\mathrm{sgn}\,\varphi,\\ R_{xi} = \mu R_{yi},\quad \mu = \mu_0\,\mathrm{sgn}\,\dot{x}_i.\end{gathered} \tag{3.8}$$

For E = 2 we have equations for two constraints: (3.3) and $\dot{x}_i = 0$ that yield relationships for $Y(\varphi), X(\varphi)$. Taking account of the rule (3.2), from (3.1) we have a system of equations for E = 2 $(\varphi \neq 0,\ R_{yi} > 0,\ |R_{xi}| < \mu_0 R_{yi}),\ I_\beta = I(1 + \beta)$:

$$\varphi^{\cdot\cdot} = f_2(M_0, W, \hat{A}, \gamma),\quad f_2 = [M_0 + z(W\sin\gamma + \hat{A}\cos\gamma)\,\mathrm{sgn}\,\varphi]\,I_\beta^{-1}; \tag{3.9}$$

$$X = x_i + z(\cos\alpha - \cos\gamma)\,\mathrm{sgn}\,\varphi,\ Y = z(\sin\gamma\,\mathrm{sgn}\,\varphi - \sin\alpha); \tag{3.10}$$

$$R_{xi} = W - mz[f_2(M_0, W, \hat{A}, \gamma)\sin\gamma + \varphi^{\cdot 2}\cos\gamma]\,\mathrm{sgn}\,\varphi; \tag{3.11}$$

$$R_{yi} = -\hat{A} + mz[f_2(M_0, W, \hat{A}, \gamma)\cos\gamma - \varphi^{\cdot 2}\sin\gamma]\,\mathrm{sgn}\,\varphi, \tag{3.12}$$

where x_i = const is the displacement of the support at the beginning of this mode of motion.

For E = 3 (two constraints) both contact points are maintained on the obstacle surface, i.e., we have the constraints equation (3.3). This yields $\varphi = Y = 0, x_i = X$, i.e., the equations for the case E = 3 ($X^{\cdot} \neq 0$, $R_{yi} > 0$, i = 1, 2) are the following:

$$X^{\cdot\cdot} = (W + \mu\hat{A})m^{-1}, \; Y = \varphi = 0; \tag{3.13}$$

$$R_{yi} = \frac{1}{2}\left[-\hat{A}(1 \mp \mu \operatorname{tg}\alpha) \mp \frac{1}{z} M_0 \sec\alpha\right]. \tag{3.14}$$

The lack of constraints is characteristic for the fourth mode. Taking (3.2) into account, we have the following system of equations for E = 4 ($y_i > 0$):

$$\varphi^{\cdot\cdot} = M_0 I^{-1}, \; X^{\cdot\cdot} = Wm^{-1}, \; Y^{\cdot\cdot} = \hat{A}m^{-1}.$$

The formula for the vertical displacement y_i of the reference point is

$$y_i = Y - z(\sin\gamma \operatorname{sgn}\varphi - \sin\alpha).$$

The systems of equations are integrated for initial and transient conditions determined as a function of the representation of the diffraction loads and the alternation of the motion modes.

4. Zero displacements and nonzero velocities determined by the impulse components dependent on the motion mode being realized are the initial conditions in approximating the effect of the diffraction phase by an instantaneous impulse (and neglect of displacements during the shock).

The impulse acting on the body is comprised of the active impulse S from the shock during the diffraction phase period and impulses of the constraint reactions. Because of the assumption about the absolutely inelastic nature of the collisions, the reactive impulse occurs at a contact point with zero vertical velocity after impact. The initial velocities $\varphi_0^{\cdot}, X_0^{\cdot}, Y_0^{\cdot}$ (after the action of the impulse) are determined by the constraints equations and the following momentum equations, in which zero velocities prior to the application of the impulse are taken into account:

$$mX_0^{\cdot} = \sum_j S_{jx}, \quad mY_0^{\cdot} = \sum_j S_{jy}, \quad I\varphi_0^{\cdot} = \sum_j \operatorname{mom}_0 S_j. \tag{4.1}$$

Here $\sum_j S_{jx}, \sum_j S_{jy}$ are the components of the principal vector of the active S and reactive impulses; $\sum_j \operatorname{mom}_0 S_j$, moment of the principal vector of the impulses with respect to the central axis z, and permanently acting forces are neglected.

Let the impulse S be collinear with the X axis and applied to the eccentricity h relative to the center of mass. The moment of the impulse is M_S = Sh. Let us consider the conditions for the origination of the first motion mode for $\varphi_0^{\cdot} > 0$. Setting $\varphi = 0, \operatorname{sgn}\varphi = 1$ while $y^{\cdot}_{01} > 0$, $y^{\cdot}_{02} = 0$ for the initial contact point velocities, we have

$$Y_0^{\cdot} = z\varphi_0^{\cdot}\cos\alpha \operatorname{sgn}\varphi \tag{4.2}$$

and the relations for the reaction impulses $S_{x1} = S_{y1} = 0$, $S_{x2} = \mu S_{y2}$, $\mu = \mu_0 \operatorname{sgn} x^{\cdot}_{02}$. From (4.1),

$$\varphi_0^{\cdot} \equiv \varphi_{0(1)}^{\cdot} = \frac{hS}{I}\left[1 - \frac{1}{2}\beta(\mu \operatorname{sgn}\varphi \sin 2\alpha - 2\cos^2\alpha)\right]^{-1} > 0, \quad \beta = \frac{mz^2}{I}; \tag{4.3}$$

$$X_0^{\cdot} \equiv X_{0(1)}^{\cdot} = \frac{S}{m} - \mu z \varphi_{0(1)}^{\cdot} \operatorname{sgn}\varphi\cos\alpha, \quad \mu = \mu_0 \operatorname{sgn} x_{0i}^{\cdot}(\varphi_0^{\cdot} > 0). \tag{4.4}$$

The inequality (4.3) is equivalent to the following [$\nu = h/(z\sin\alpha) > -1$]:

$$\mu_0 < \lambda_1 \equiv 2(1 + \beta\cos^2\alpha)/(\beta\sin 2\alpha) > 0(\nu > 0), \; \mu_0 > \lambda_1(\nu < 0). \tag{4.5}$$

For the contact point velocity we have ($\mu = \mu_0 \operatorname{sgn} x^{\cdot}_{02}$),

$$x_{02}^{\cdot} = X_0^{\cdot} - z\varphi_0^{\cdot}\sin\alpha = \frac{S}{m}\left\{1 - 2\beta\nu\left(\sin^2\alpha + \frac{1}{2}\mu\sin 2\alpha\right)[2(1+\beta\cos^2\alpha) - \beta\mu\sin 2\alpha]^{-1}\right\},$$

where, according to (4.5), the denominator of the fraction is positive for $\nu > 0$ and negative for $\nu < 0$ for the case $x^{\cdot}_{02} > 0$. If $x^{\cdot}_{02} < 0$, then the denominator is nonnegative for $\nu > 0$, and it can be shown that conditions sufficient for the realization of the first mode of motion for $\dot{\varphi}_0 > 0$ are $[\lambda_0 \equiv \lambda_1 - \nu \tan \alpha/(\nu + 1)]$:

$$\begin{aligned} &\mu_0 < \lambda_0 < \lambda_1, \quad 0 < \nu < \lambda_1 \operatorname{ctg} \alpha \ldots \operatorname{sgn} \dot{x}_{02} = 1, \\ &\mu_0 > \lambda_0 > \lambda_1, \quad -1 < \nu < 0 \ldots \operatorname{sgn} \dot{x}_{02} = 1, \end{aligned} \tag{4.6}$$

$$\left.\begin{aligned} &\lambda_0 < 0, \quad \nu > \lambda_1 \operatorname{ctg} \alpha \\ &\mu_0 < |\lambda_0| < \lambda_i \quad \text{or} \quad \mu_0 < \lambda_i < |\lambda_0| \end{aligned}\right\} \ldots \operatorname{sgn} \dot{x}_{02} = -1.$$

For real values of α and β the parameter $\lambda_1 > 1$ and the domain $\mu_0 > \lambda_0 > \lambda_1$, $-1 < \nu < 0$ (4.6) correspond to the value of the coefficient $\mu_0 > 1$. The condition $\mu_0 < \lambda_1 < |\lambda_0|$ for $\lambda_0 < 0$ means, in practice, that $\mu_0 < 1$.

For the first motion mode to occur for $\dot{\varphi}_0 < 0$, by taking $\varphi = 0$, $\operatorname{sgn} \varphi = -1$, $y^{\cdot}_{01} = 0$, $y^{\cdot}_{02} > 0$, $S_{x1} = \mu S_{y1}$, $S_{x2} = S_{y2} = 0$, we obtain (4.2)-(4.4) as well as $\dot{x}_{01} = \dot{X}_0 - z\dot{\varphi}_0 \sin \alpha$. It is seen from (4.3) that the inequality $\dot{\varphi}_0 < 0$ is satisfied for any μ_0 and $\nu < 0$. For the velocity of the contact point we hence have

$$\dot{x}_{01} = \frac{S}{m}\left\{1 - 2\beta\nu\left(\sin^2 \alpha - \frac{1}{2}\mu_0 \sin 2\alpha\right)\left[2(1 + \beta\cos^2 \alpha) + \beta\mu_0 \sin 2\alpha\right]^{-1}\right\}.$$

For $\mu_0 > -\lambda_0$ the inequality $x^{\cdot}_{01} > 0$ is satisfied automatically also for any μ_0 since $\lambda_0 > 0$. Therefore, the first motion mode with the initial velocities (4.3) and (4.4) is realized under the conditions

$$\begin{aligned} &\mu_0 < |\lambda_0| < 1, \\ &0 < \nu < \lambda_1 \operatorname{ctg} \alpha \quad (\operatorname{sgn} \varphi = 1, \ \operatorname{sgn} \dot{x}_{02} = 1), \\ &\nu > \lambda_1 \operatorname{ctg} \alpha \quad (\operatorname{sgn} \varphi = 1, \ \operatorname{sgn} \dot{x}_{02} = 1), \\ &-1 < \nu < 0 \quad (\operatorname{sgn} \varphi = -1, \ \operatorname{sgn} \dot{x}_{01} = 1). \end{aligned}$$

Let us consider the conditions for origination of the second mode, for which $S_{x1} = S_{y1} = x^{\cdot}_{02} = y^{\cdot}_{02} = 0$, i.e., we have (4.2) and $\dot{X}_0 \equiv \dot{X}_{0(2)} = z\dot{\varphi}_0 \sin \alpha$. Substituting the values of these velocities $X^{\cdot}_0$ and $Y^{\cdot}_0$ into (4.1), we obtain

$$\dot{\varphi}_0 \equiv \dot{\varphi}_{0(2)} = hS(\nu + 1)/[I\nu(\beta + 1)]. \tag{4.7}$$

Realization of this mode is possible under the conditions

$$\dot{\varphi}_0 > 0, \quad |S_{x2}| < \mu_0 S_{y2}, \tag{4.8}$$

where, as is seen from (4.7), the former is always satisfied ($\nu > -1$).

We write for the reaction impulses

$$\begin{aligned} S_{x2} &= S - mz \sin \alpha \cdot S(h + z \sin \alpha)/I_\beta \\ S_{y2} &= mz \cos \alpha \cdot S(h + z \sin \alpha)/I_\beta. \end{aligned}$$

In the case $S_{x2} > 0$ the second inequality in (4.8) yields $\mu_0 > \lambda_0$ for $\nu < \lambda_1$ cotan α, where there should be $\nu > 0$ in order to have $\mu_0 < 1$, while if $S_{x2} < 0$, then $\mu_0 > -\lambda_0$ for $\nu > \lambda_1 \cdot$ cotan α, $\lambda_0 < 0$. On the line $\nu = \lambda_1$ cotan α we have $S_{x2} = 0$. Therefore, the second motion mode with the initial angular velocity from (4.7) is realized when $|\lambda_0| < \mu_0 < 1$, $\nu > 0$.

Origination of the third mode is possible for $\dot{Y}_0 = \dot{\varphi}_0 = 0$, $\dot{X}_0 > 0$. Equations (4.1) yield $\sum_i S_{xi} > 0$, $\sum_i S_{yi} = \sum_i \operatorname{mom}_0 S_i = 0$, i.e., $S_{y1} + S_{y2} = 0$ and, according to Coulomb's law, $S_{x1} + S_{x2} = 0$. Hence, at the beginning of slip no horizontal reactive impulses occur and the initial velocity is determined (without losses because of collisions) by the total active impulse $X^{\cdot}_0 = S/m$.

From the equation for the balance of the moment of the impulse $hS + z(S_{y1} - S_{y2}) \cos \alpha = 0$, i.e., taking into account that $S_{y1} + S_{y2} = 0$, we obtain values of the vertical impulses $S_{y1} = -S_{y2} = -hS/(2z \cos \alpha)$. This result does not contradict the unilateral properties of the constraints just for $h = 0$, i.e., the mode 3 occurs under the unique condition $\nu = 0$, where the initial velocity corresponds to a body completely free of constraints.

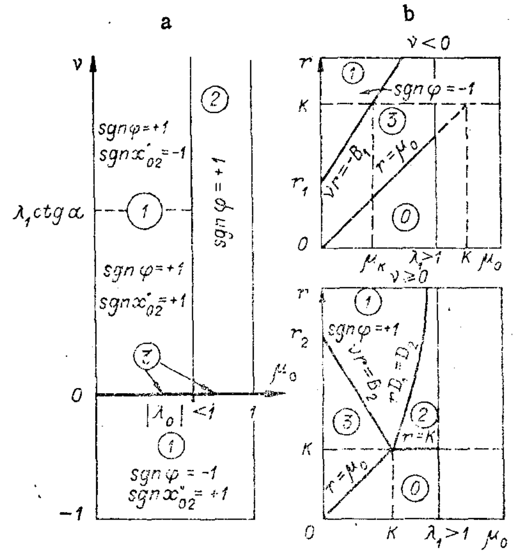

Fig. 2

The case of the origination of different modes of motion under the action of an instantaneous impulse is shown in Fig. 2a in the $\mu_0 - \nu$ plane.

5. Let us consider the motion conditions by representing the diffraction phase by a process of finite duration in which the maximal loads along the X axis correspond to the time $t = 0$. In particular, this permits clarification of the condition of body immobility for low pressures Δp_f when the loads from the shock are equilibrated statically. In this case, three constraints $(X = Y = \varphi = 0)$ are imposed on the body and the equilibrium equations have the form

$$M_0 - z[(R_{y_2} - R_{y1}) \cos \alpha - R_x \sin \alpha] = 0,$$
$$R_x \equiv R_{x1} + R_{x2} = W,\ R_y \equiv R_{y1} + R_{y2} = -\hat{A}.$$

Hence we obtain a formula for the vertical reactions

$$R_{yi} = -\frac{1}{2}\left(\hat{A} \pm W \operatorname{tg} \alpha \pm \frac{M_0}{z} \sec \alpha\right).$$

By virtue of the unilateral nature of the vertical constraints and the Coulomb law, the immobility conditions are $R_{yi} > 0$, $R_x < \mu_0 R_y$, or, after substitution of the load values for $t = 0$ from Sec. 2, $M_0 = M_m$, $W = W_m$, $A = 0$, $r < K$, $r < \mu_0$, $K = \cot\alpha/(\nu + 1)$, $r = M_m/(mgh^\circ)$, $\nu = h^\circ/(z \sin \alpha)$, $h^\circ = M_m/W_m$.

Furthermore, we consider the conditions of displaceability at the time $t = +0$ for which we take $X = Y = \varphi = X^{\cdot} = Y^{\cdot} = \varphi^{\cdot} = 0$. We assume that for $h^\circ < 0$, $\alpha < \pi/4$, and the value $\mu_0 < \lambda_1$. The quantity $\lambda_1 > 1$ and the coefficient μ_0 can be variated to 1. The parameter α can have the value $\alpha > \pi/4$ for $h^\circ > 0$.

For mode 1 to occur for $\varphi = 1$ then there should be $\varphi^{\cdot\cdot} > 0$ while the horizontal acceleration of the center of inertia should exceed the acceleration that would occur without slip, i.e., by virtue of (3.10) there should be $X^{\cdot\cdot}(0) > z\varphi^{\cdot\cdot}(0) \sin \alpha$. Expanding the inequalities by using the equations for the first mode in which we take $\mu = \mu_0$, $\operatorname{sgn} \varphi = 1$ we obtain

$$rD_1 > D_2,\ \nu r > B_2(\nu > 0);\ rD_1 > D_2,\ \mu_0 > K\ (\nu = 0);$$
$$D_1 = 1 + \beta[1 - (\nu + 1)(\sin^2 \alpha + (1/2)\mu_0 \sin 2\alpha)];$$
$$D_2 = \mu_0(1 + \beta \sin^2 \alpha) - (1/2)\beta \sin 2\alpha;\ B_2 = \operatorname{ctg} \alpha - \mu_0.$$

For the reverse rotation, by setting $\varphi^{\cdot\cdot} < 0$, $\ddot{x}_{01} > 0$ for $\operatorname{sgn} \varphi = -1$ we obtain $-\nu r > B_1 \equiv \cot\alpha + \mu_0 (\nu < 0)$ from the same equations. This condition is valid although the denominator in the right side of (3.8) remains positive, i.e., for $\mu_0 > \lambda_1$.

Rotational motion around the right support point occurs for $\varphi^{\cdot\cdot} > 0$, $R_{x2} < \mu_0 R_{y2}$, $R_{y2} > 0$. Expanding these inequalities by using (3.9)-(3.12), for $\varphi = 1$ we will have $rD_1 < D_2$, $r > K\ (\nu \geq 0)$.

Slip (mode 3) is realized for $X^{\cdot\cdot} > 0$, $R_{yi} > 0$. By using (3.13) and (3.14), we obtain the conditions for realizing this mode

$$\mu_0 < r < B_2/\nu(\nu > 0),\ r > \mu_0,\ \mu_0 < K(\nu = 0),\ \mu_0 < r < B_1/|\nu|\,(\nu < 0).$$

The displaceability conditions in the μ_0–r plane are shown in Fig. 2b by domains whose numbers correspond to the motion modes, while the boundaries are determined by the parameters K, ν, and β, where K determines the domain of immobility <0> and is an index of the tendency of the body to a definite mode of motion. As K grows, the immobility domain broadens, while the tendency to instability diminishes. The vertical boundary $\mu_0 = \lambda_1$ for the initial values of the parameters overlaps the quantity $\mu_0 = 1$. For $\nu < 0$ and $\alpha < \pi/4$, we have $K > 1$.

6. Let us determine the transient conditions. The passage from mode 1 to mode 2 (1–2) is realized when retarding the contact point, which fixes the change in the sign of the velocity for the numerical solution, i.e., the condition $x^{\cdot}_{0-}x^{\cdot}_{0} < 0$, where $x^{\cdot}_{0-}$ is the velocity in the last time spacing (for numerical integration of the systems of equations). The transition 2–1 is related to the excess of the horizontal reaction of the (Coulomb) limit value according to the condition $R_{xi} > \mu_0 R_{yi}$. The transitions 3–1 are determined when the negative sign appears for one of the vertical reactions $R_{yi} < 0$, while 1–4 and 2–4 are the negative sign of the reaction R_{yi} for a unique contact point. The mentioned transitions are shock-free.

Let us consider the shock transitions 1–3, 2–3, 4–1, 4–2, and also 1–1, 1–2, 2–1, 2–2 that are associated with a change in the sign of the angle φ. A criterion of the transitions 4–1, 4–2 is the disappearance of the distance y_i between one of the support points and the ob stacle. As the motion goes from one mode to another not associated with the impacts of the reference points on the obstacle, the displacements and velocities remain continuous. Constraints are suddenly imposed on the moving body under impact transitions, whereupon reactive impulses occur. As above, a collision is considered instantaneous and absolutely inelastic (the coefficient of restoration equals zero); consequently, displacements and impulses from the permanently acting forces are neglected. The relationship between the reactive impulse components correspond to the Coulomb law with coefficient μ_0. The system momentum changes (diminishes) as a result of the action of the reactive impulses S_x and S_y during impact, i.e., a jump diminution in the velocities or a transformation of the motion occurs.

Let us consider the case of body incidence at an angle $\varphi \neq 0$ on one of the supports at the time $t = t_*$. Before impact ($t = -t_*$), the velocities had the values $X^{\cdot}$, $Y^{\cdot}$, $\varphi^{\cdot}$ and the values $X^{\cdot}_*$, $Y^{\cdot}_*$, and $\varphi^{\cdot}_*$ after the impact ($t = +t_*$). The equations of the moments of the impulses and the impulses with the inelastic impact coupling equation $y^{\cdot}_{0i}(+t_*) = 0$

$$I\left(\varphi^{\cdot}_* - \varphi^{\cdot}\right) = z\left(S_x \sin\gamma - S_y \cos\gamma \operatorname{sgn}\varphi\right),$$
$$m\left(X^{\cdot}_* - X^{\cdot}\right) = -S_x,\quad m\left(Y^{\cdot}_* - Y^{\cdot}\right) = S_y,\quad Y^{\cdot}_* = z\varphi^{\cdot}_* \cos\gamma \operatorname{sgn}\varphi \tag{6.1}$$

contain five desired quantities $S_x, S_y, \varphi^{\cdot}_*, X^{\cdot}_*, Y^{\cdot}_*$ where the reactive impulses can be expressed in terms of the angular velocity $\varphi^{\cdot}_*$

$$S_x = \frac{I \operatorname{sgn}\varphi}{z \sin\gamma}\left[\varphi^{\cdot}_*(1 + \beta\cos^2\gamma) - \varphi^{\cdot} - \frac{\beta}{z} Y^{\cdot} \cos\gamma \operatorname{sgn}\varphi\right]; \tag{6.2}$$
$$S_y = m\left(z\varphi^{\cdot}_* \cos\gamma \operatorname{sgn}\varphi - Y^{\cdot}\right).$$

These expressions are valid in cases of the origination of modes 1 or 2. Let us add a relationship dependent on the mode to be expected. If the mode 2 occurs after impact, then $x^{\cdot}_{0i}(+t_*) = 0$, or

$$X^{\cdot}_* = z\varphi^{\cdot}_* \sin\gamma \operatorname{sgn}\varphi \quad (\mu_0 > |\lambda|,\ \lambda = S_x/S_y)$$

and we obtain from (6.1)

$$\varphi^{\cdot}_* = \frac{1}{\beta + 1}\left[\varphi^{\cdot} + \frac{\beta}{z}(X^{\cdot} \sin\gamma + Y^{\cdot} \cos\gamma) \operatorname{sgn}\varphi\right].$$

If it turns out in the verification that the condition $\mu_0 > |\lambda|$ is not satisfied, then mode 1 occurs as a result of the impact, where

$$S_x = \mu_0 S_y \operatorname{sgn} \dot{x}_{0i}, \quad \dot{x}_{0i} = X^{\cdot} - z\varphi^{\cdot} \sin\gamma \operatorname{sgn}\varphi,$$

and, taking account of (6.1) and (6.2), we obtain the following formulas for the velocities:

$$X^{\cdot}_* \equiv X^{\cdot\prime}_* = X^{\cdot} - \mu_0 \left(z\varphi^{\cdot\prime}_* \cos\gamma \operatorname{sgn}\varphi - Y^{\cdot}\right) \operatorname{sgn} \dot{x}_{0i},$$

$$\varphi^{\cdot}_* \equiv \varphi^{\cdot\prime}_* = \frac{\varphi^{\cdot} - \frac{\beta}{z} Y^{\cdot} \left(\mu_0 \sin\gamma \operatorname{sgn} \dot{x}_{0i} - \cos\gamma\right) \operatorname{sgn}\varphi}{1 + \frac{1}{2}\beta\left(1 + \cos 2\gamma - \mu_0 \sin 2\gamma \operatorname{sgn} \dot{x}_{0i}\right)}.$$

Furthermore, we consider the use of shock transitions with a change in the sign of the angle φ (tilting). Here the impulse occurs under the support which had no contact with the obstacle prior to the impact (otherwise the motion ceases because of inelastic impact). It hence follows that the formulas presented are valid even for this case if we set $\gamma = \alpha$ therein and take the value of $\operatorname{sgn}\varphi$ for $t = +t_*$, i.e., for the mode of motion again occurring. The signs of the velocities calculated by means of the formulas mentioned should correspond to the modes of motion that appear after impact. Without such correspondence, slip (mode 3) or braking (halting) of the body will evidently occur.

If rotation does not occur after impact, then the presence of impulses under both supports must be allowed. Introducing the four impulses $S_{y1} > 0$, $S_{y2} > 0$, S_{x1}, and S_{x2} by assuming that $\varphi^{\cdot}_* = Y^{\cdot}_* = 0$ after impact ($t = +t_*$), and using the notation $S_x = S_{x1} + S_{x2}$, we write the equations for impact with the transition to slip for $\gamma = \alpha$

$$-I\varphi^{\cdot} = z[(S_{y1} - S_{y2}) \cos\alpha + S_x \sin\alpha],$$

$$m\left(X^{\cdot}_* - X^{\cdot}\right) = -S_x, \quad -mY^{\cdot} = S_{y1} + S_{y2}, \quad S_x = \mu_0 (S_{y1} + S_{y2}) \operatorname{sgn} X^{\cdot}_*.$$

We hence have ($i = 1, 2$)

$$S_x = -\mu_0 m Y^{\cdot} \operatorname{sgn} X^{\cdot}_*; \tag{6.3}$$

$$S_{yi} = \mp(I\varphi^{\cdot} \pm mzY^{\cdot} \cos\alpha + zS_x \sin\alpha)/(2z\cos\alpha); \tag{6.4}$$

$$X^{\cdot}_* = \mu_0 Y^{\cdot} \operatorname{sgn} X^{\cdot}_* + X^{\cdot}. \tag{6.5}$$

Let $X^{\cdot}_* > 0$ ($\operatorname{sgn} X^{\cdot}_* = 1$). Then taking into account that $Y^{\cdot} < 0$, we obtain from (6.5)

$$\mu_0 < -X^{\cdot}/Y^{\cdot}, \tag{6.6}$$

and from (6.3) the expression

$$S^{\cdot}_x = -\mu_0 m Y^{\cdot} > 0. \tag{6.7}$$

Let us note that, in the numerical realization of the problem, the impact criterion is a change in the product $\operatorname{sgn}\varphi(t_i)\operatorname{sgn}\varphi(t_{i+1})$ from plus to minus in the i + 1 time step ($t_i < t_* < t_{i+1}$). Let us introduce $\operatorname{sgn}\varphi_* \equiv \operatorname{sgn}\varphi(t_{i+1}) = -\operatorname{sgn}\varphi(t_i)$ to fix the change in the mode of the motion. Using the function $\operatorname{sgn}\varphi_*$ for $t = +t_*$, and taking account of (6.7), we eliminate S_x and the velocity $Y^{\cdot} = -z\varphi^{\cdot} \operatorname{sgn}\varphi_* \cos\alpha$ from (6.4) to obtain

$$S_{yi} = I\varphi^{\cdot}\left[\beta \operatorname{sgn}\varphi_* \left(\cos^2\alpha \mp \frac{1}{2}\mu_0 \sin 2\alpha\right) \mp 1\right] \Big/ (2z\cos\alpha).$$

Expanding the inequality $S_{yi} > 0$, and taking into account that the transition to slip is $\varphi^{\cdot} \operatorname{sgn}\varphi < 0$ ($\varphi^{\cdot} \operatorname{sgn}\varphi_* > 0$) for both variants, we obtain the condition

$$\mu_0 < \lambda^0, \quad \lambda^0 = 2(\beta\cos^2\alpha - \operatorname{sgn}\varphi_*)/(\beta \sin 2\alpha).$$

Now if we assume that $X^{\cdot}_* < 0$, then by repeating the procedure we will have, instead of (6.6),

$$\mu_0 < X^{\cdot}/Y^{\cdot}, \tag{6.8}$$

and the following expression for the vertical impulses

$$S_{yi} = I\varphi^{\cdot}\left[\beta \operatorname{sgn}\varphi_* \left(\cos^2\alpha \pm (1/2)\mu_0 \sin 2\alpha\right) \mp 1\right] / (2z\cos\alpha).$$

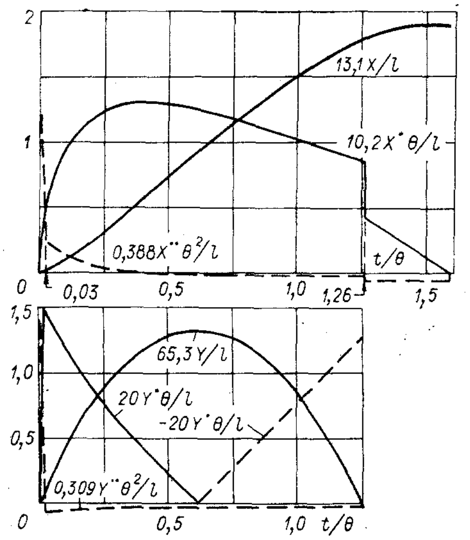

Fig. 3

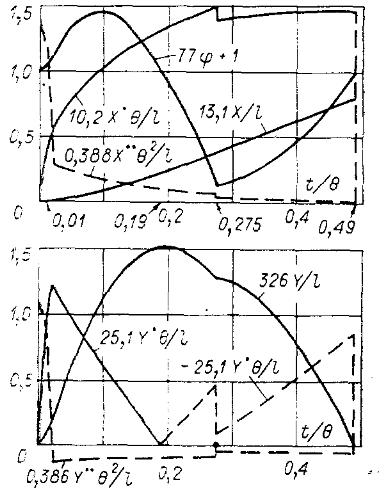

Fig. 4

We now obtain from the inequalities $S_{yi} > 0$

$$2(-\beta\cos^2\alpha + \operatorname{sgn}\varphi_*)/(\beta\sin 2\alpha) < \mu_0 < 2(\beta\cos^2\alpha + \operatorname{sgn}\varphi_*)/(\beta\sin 2\alpha) \equiv \lambda^{00}.$$

For real values of α and β the left inequality is satisfied automatically and it is sufficient to write $\mu_0 < \lambda^{00}$. Since $Y^{\bullet} < 0$ always before impact, while $\mu_0 > 0$, then the inequalities (6.6) and (6.8) are satisfied only under the condition $\operatorname{sgn} X^{\bullet} = \operatorname{sgn} X^{\bullet}_{*}$. This permits extending the results to the form

$$\mu_0 < |X^{\bullet}/Y^{\bullet}|, \quad \mu_0 < \varkappa \equiv 2(\beta\cos^2\alpha - \operatorname{sgn} X^{\bullet}\operatorname{sgn}\varphi_*)/(\beta\sin 2\alpha), \tag{6.9}$$

where, as before, the sign of the quantity φ_* is opposite to the sign of the angle φ before impact, while the velocity after impact with the passage to the mode 3 is determined by (6.5) in which $\operatorname{sgn} X^{\bullet}_{*}$ should be replaced by $\operatorname{sgn} X^{\bullet}$.

The body ceases to move when condition (6.9) is not satisfied. The loss of stability is determined by compliance with the condition $|\gamma| > \pi/2$.

The problem formulated is programmed for the ES electronic computer. The system of equations is integrated by the Runge-Kutta method.

7. Examples are presented of analysis of the motion of a body in the shape of a parallelepiped, loaded by a shock with the parameters $\Delta p_f = 0.1$ MPa, and duration of the positive phase of the velocity head $\theta = 0.065$ sec. The wave parameters (Δp_{vel}, Δp_{ref}, D_f, $f(t/\theta)$) are determined completely by the pressure Δp_f [2]. Two parallelepiped models of identical size were examined, of height l_y, length l_z, width (streamwise dimension) B with supports specifying the distance H_0 from the lower face of the model to the obstacle: $B/l = 0.707$, $H_0/l = 0.156$, $l_y/l = 0.448$, $l^2/F_\delta = l_y l_z/(B l_z) = 0.633$, l^2 is the planform area of the body, center of inertia is on the vertical axis of symmetry, and for body 1 is below, and for 2 above the geometric center. The coordinates of the center of inertia of body 1 are $\alpha = 0.606$ rad, $z/l = 0.43$, and for body 2 are $\alpha = 0.903$ rad, $z/l = 0.571$. The functions $\hat{C}$, $\alpha_{W,A,M}$ (see Sec. 1) are obtained by model testing in a wind tunnel. The impulse components of the diffraction phase are taken from data in [5]: $K_W \equiv S_W/S_* = 1.55$, $\gamma_A \equiv S_A/S_W = 0.55$, $\chi_0 \equiv M_S/(l S_W) = 0.35$ rad for model 1, and 0.14 for model 2, $S_* = (3B/2D_f)(\Delta p_{ref} F_\delta + Q(0))$ is the characteristic impulse, D_f is the wave front velocity, $\Delta p_f l^2/(mg) = 16.1$, $g\theta^2/l = 0.195$, $\mu_0 = 0.67$, $g = 9.81$ m/sec². The parameter $\beta = mz^2/I = 3.24$ for model 1, and 5.47 for model 2.

Graphs of the horizontal and vertical motion of body 1 are presented in Fig. 3. Significant acceleration gradients are observed in the diffraction phase at whose end the vertical acceleration becomes negative. For $t/\theta \leq 0.03$, body rotation occurs around the right support (mode 2). For $0.03 < t/\theta \leq 1.26$, slip with rotation occurs (mode 1), and then the vertical velocity vanishes upon collision of the left support with the obstacle, the horizontal velocity is halved, and the body makes the transition into slip for $\varphi = 0$ (mode 3). The maximum velocity $X^{\bullet}_m\theta/l = 0.132$ is achieved at the time $t/\theta = 0.42$, i.e., braking starts long before termination of the action of the shock. The maximum of the vertical shift in the center of inertia $Y_m/l = 0.02$ is reached at the time $t/\theta = 0.61$. Complete braking (halting) occurs for $t/\theta = 1.6$, when the horizontal shift reaches the maximum $X_m/l = 0.146$.

The motion parameters of body 2 are given in Fig. 4. Here the body moves according to mode 2 $(\varphi > 0)$, in the interval $0 < t/\theta \leq 0.01$, and then loses contact with the obstacle (passage to mode 4), where the sign of the angle φ changes at the time $t/\theta = 0.19$. Impact on the obstacle for $\varphi < 0$, $t/\theta = 0.275$ results in a velocity drop and passage to mode 2. Collision of the second support with the obstacle for $t/\theta = 0.49$ and $X_m/l = 0.061$ causes halting of the body. Therefore, a change in the coordinates of the center of inertia would result in a substantial difference in both the qualitative pattern of body displacement near the obstacle and in the quantitative indices of the motion parameters. This distinction is governed to a significant degree by the change in the moment of the impulse and the aerodynamic forces relative to the center as well as the reaction of the bonds with the shift in the center of inertia.

Computations were performed for the version of approximating the diffraction phase effect according to the relationships in Sec. 5.

8. Let us give an estimate of the possible displacements of the surface of the obstacle bounding a linearly elastic half-space with the acoustic resistance A_1 (for instance, soil). Taking the boundary pressure in the form $\Delta p(t) = \Delta p_f(1 - t/\theta)^n$, we have a formula for the surface displacement u(t) in a one-dimensional approximation

$$U \equiv u(t)A_1(n+1)/(\Delta p_f\theta) = 1 - (1 - t/\theta)^{n+1}\ (0 \leqslant t/\theta \leqslant 1),\ U = 1\ (t/\theta \geqslant 1).$$

The maximum displacement to the end of the action of the pressure equals

$$u_m = \Delta p_f\,\theta/[A_1(n+1)].$$

Taking $A_1 = 1.6 \cdot 10^6$ kg/(m²·sec) (medium-density soil), $\Delta p_f = 0.1$ MPa, $\theta = 0.065$ sec, $n = 2$, we obtain $u_m/l = 6.4 \cdot 10^{-3}$. Taking account of the value of the maximum displacements mentioned in Sec. 7, we arrive at the conclusion that the surface displacement is negligibly small, i.e., a hypothesis as about a solid obstacle is allowable for the surface of soil in this case.

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